CFD-Simulation thermoakustischer Resonanzeffekte zur Bestimmung der Flammentransferfunktion

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Master Thesis

- Master Thesis at Technische Universität Berlin
  - Supervision by the “Fachgebiet für Experimentelle Strömungsmechanik“
  - Investigation of thermoacoustics in combustion systems by Jun.-Prof. Dr. J. P. Moeck
  - Determine Flame Transfer Functions (FTF) amongst other things

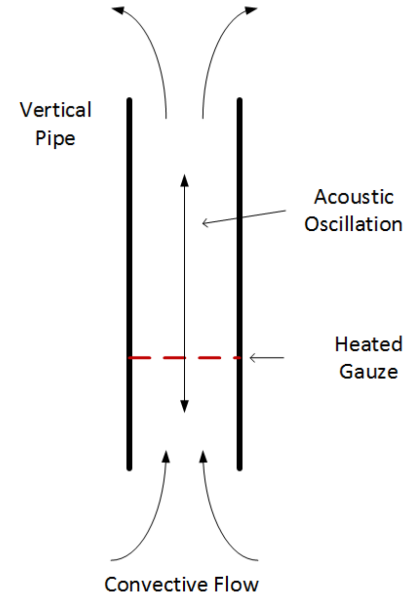
- Content of the Master Thesis
  - “Determination of Heat Transfer Functions with 3D CFD and their application on 1D CFD”
  - Simulation of thermoacoustics in a Rijke Tube with ANSYS CFX
  - Finding a model to represent the effects of heated wires in a fluctuating flow
  - Determination of a simple mathematical model
  - Simulate a Transfer Function for the heated wires, from the idea of FTFs

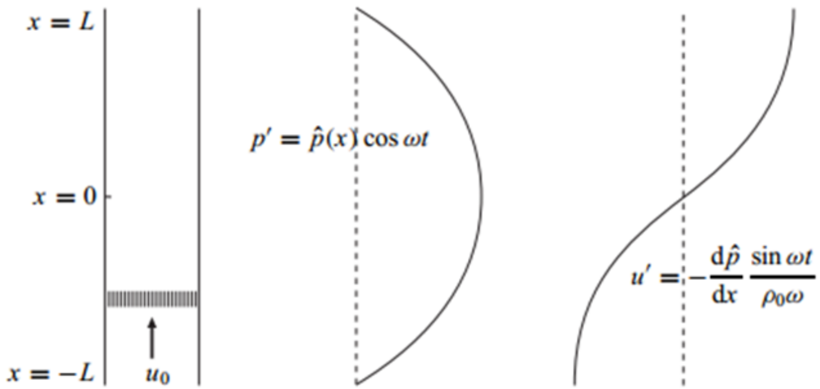
- Thesis was written in cooperation with CFX Berlin

The Rijke Tube

- Singing Tube (19th century)
  - Pipe with a compact heat source in it which generates an audible sound
  - Heat source inducts convective flow followed by an acoustic disturbance
  - Reflection at the pipe ends leads to an acoustic oscillation
  - Acoustics disturbs heat source, fluctuating heat source disturbs acoustics again
  - Feedback loop
  - Standing wave at the pipes resonance frequency self excites
  - 0.4 m long pipe ~ 500 Hz

⇒ Thermoacoustic resonator





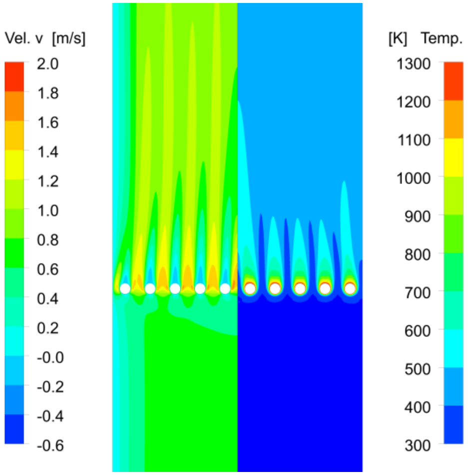
Motivation

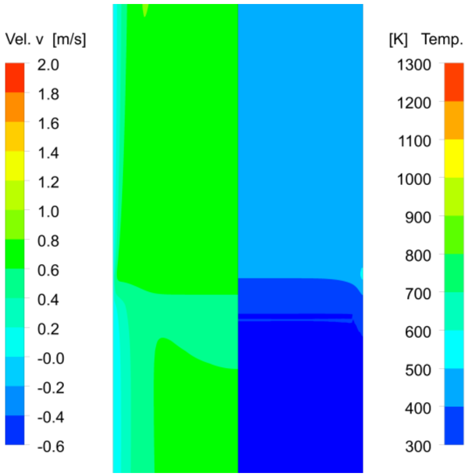
- Reduction of the simulation time of thermoacoustics
  - **Simulation** of thermoacoustic effects could replace cost intensive experiments
  - **Reduce** complex 3D physical problem into 1D mathematical model
  - **Modeling** the heat release rate in a fluctuating stream

⇒ No wires needed to be simulated

⇒ Lesser mesh elements needed

⇒ Less computational power needed





Rijke Tube in ANSYS CFX

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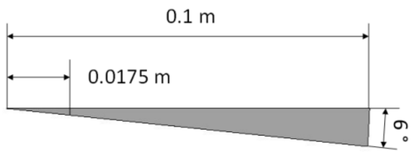
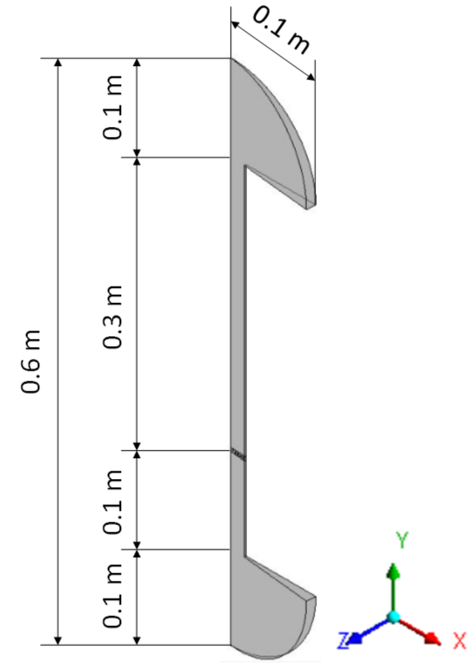
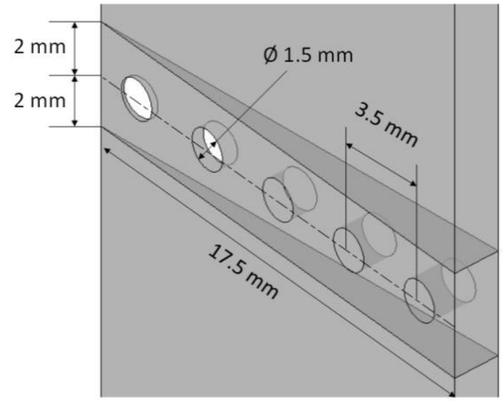
- Geometry
  - A section of 6° of a vertical pipe
  - Length: 0.4 m, Diameter: 0.035 m
  - Ambient volumes at the top and bottom with a radius of 0.1 m
  - 5 heated wires at ¼ of the pipe length with a diam. of 1.5 mm





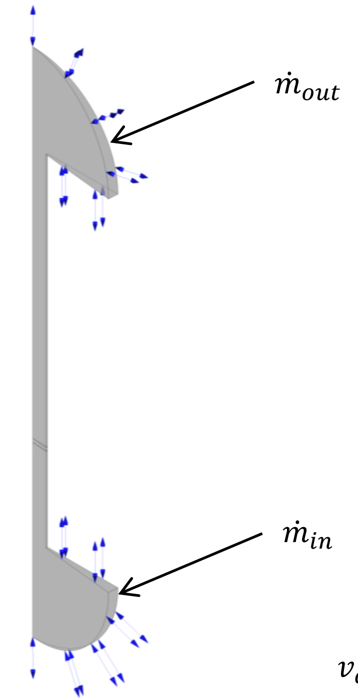
- Fluid domain setup
  - Compressible air from ANSYS CFX database
  - Heat transfer model: Total energy
  - Turbulence model: (none) laminar
  - Buoyancy model
    - Buoyant
    - Gravity Y Dirn.: -g
    - Ref. density: $\rho_{ref} = P_{ref} \cdot \frac{M_{air}}{R \cdot T_{ref}}$

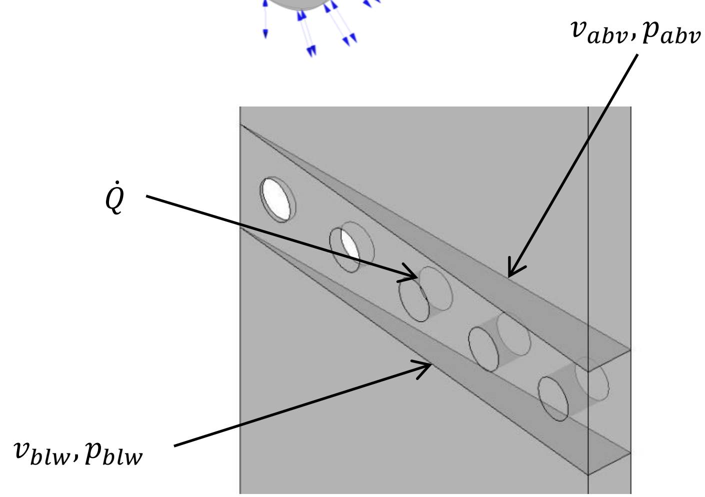


Rijke Tube ANSYS CFX

- Boundary conditions
  - Pipe wall: Default „no slip“ Wall
  - Symmetry planes: Symmetry condition
  - Ambient volumes surfaces: Opening condition
    - At ambient pressure: 1 atm
    - Opening temperature: 300 K
  - Wires surfaces
    - „No slip“ Wall
    - Fixed temperature: 1300 K, 1800K and 2300 K

- Solver setup
  - Default settings
  - MAX residual < 10^{-4}





Simulating the Rijke Tube

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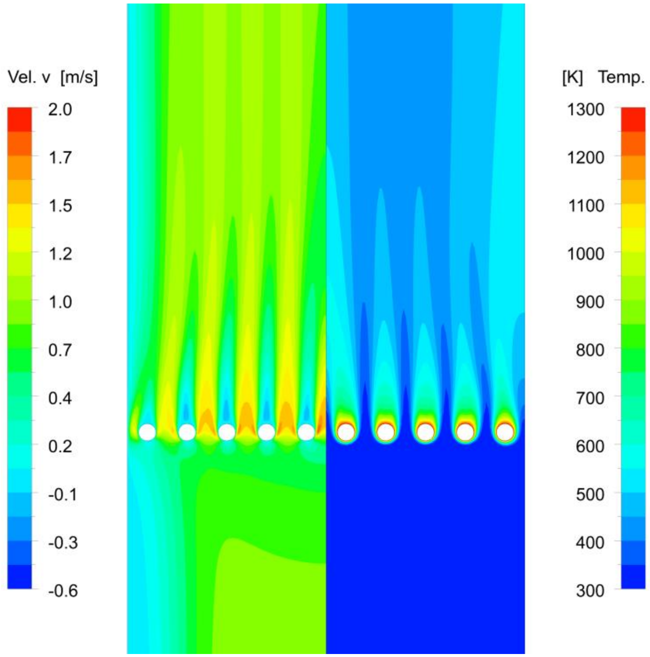
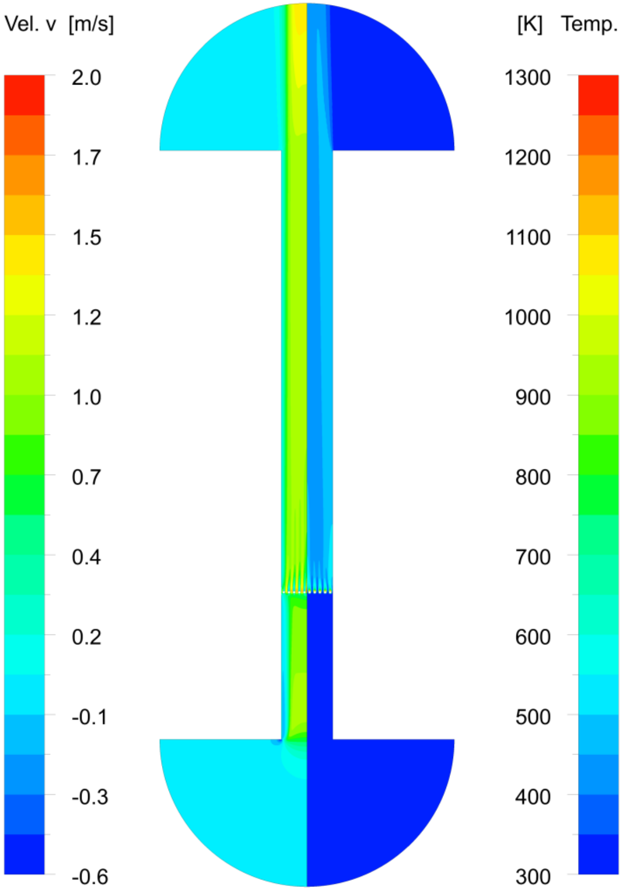
- Steady-state simulation with 1300 K wire temperature
  - Velocity: $v_{blw} \approx 0.5 \text{ m/s}$, $v_{abv} \approx 0.75 \text{ m/s}$
  - Temperature: $T_{blw} = 300 \text{ K}$, $T_{abv} \approx 464 \text{ K}$
  - Mass flow rate: $\dot{m} \approx 9.25 \cdot 10^{-6} \text{ kg/s}$





Transient Setup in ANSYS CFX

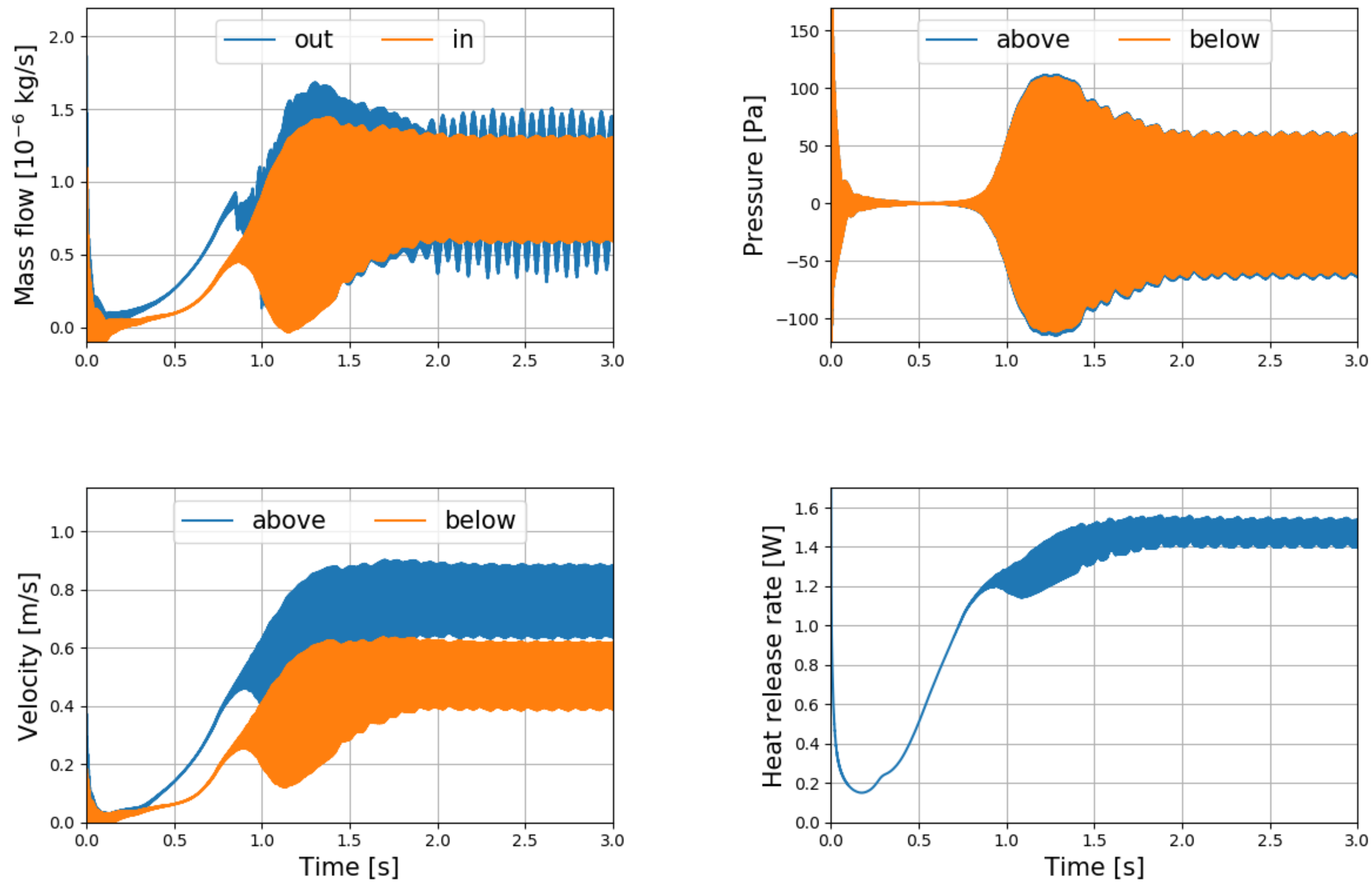
CFX BERLIN

- Solver Setup
  - Default settings
  - Coefficient Loops: min. 2, max.15
  - Max Residual: 10^{-3}
- Calculation of the time step size
  - Resonance frequency f_1 was calculated considering the temperature jump through the heat source beforehand
  - $\Delta t = \frac{1}{f_1} \cdot \frac{1}{50}$
  - $\Delta t = 38\ \mu s$ for all simulations

Parameter	Value				Unit
$\Delta \boldsymbol{T}$	300	1000	1500	2000	K
f_1	434	489	511	529	Hz
Δt	-	46	41	38	µs

Transient Simulation of the Rijke Tube

- Start from Ambient Conditions with 1300 K wire temperature

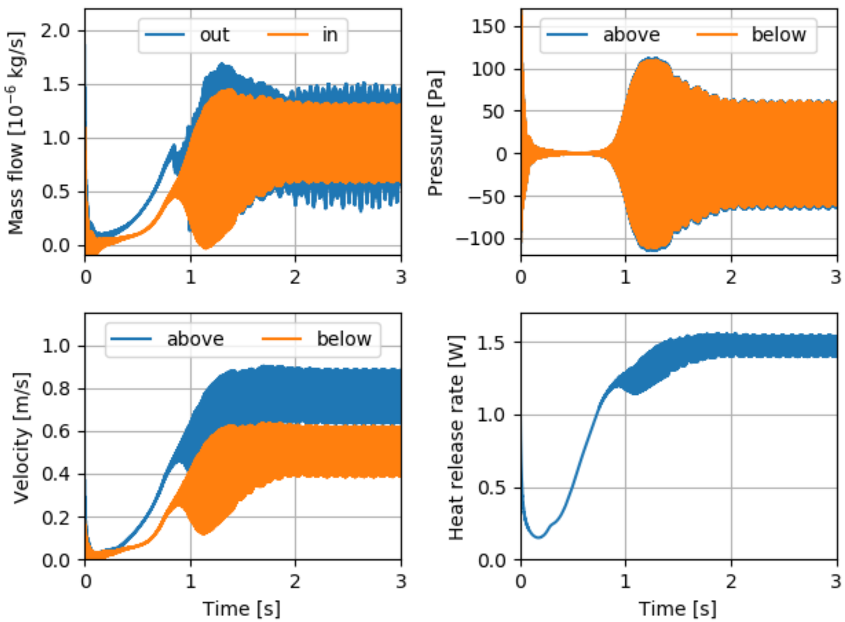


Transient Simulation of the Rijke Tube

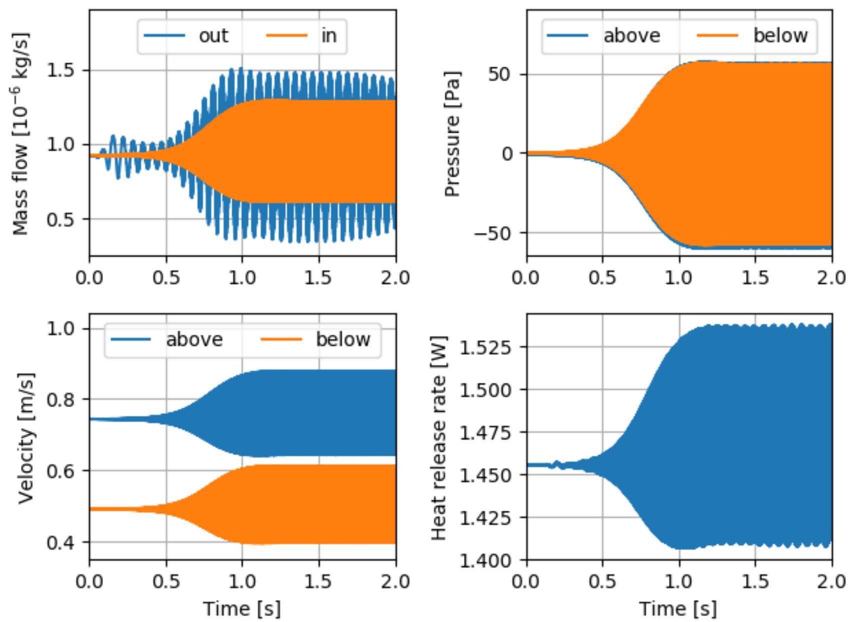
CFX BERLIN

- Start from steady-state solution
  - Wire temperature of 1300 K
  - Oscillation starts immediately
  - Limit cycle is reached 1 s earlier
  - ANSYS CFX can represent thermoacoustic oscillations
  - 3 s simulated time ⇒ 3 day real time at 4 CPU Cores

Start from ambient conditions



Start with steady-state solution

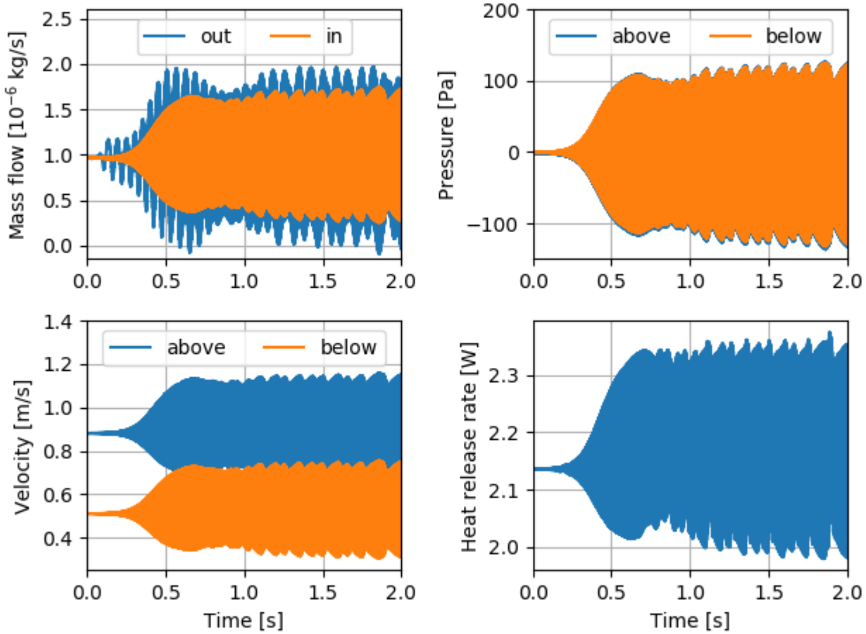


Transient Simulation of the Rijke Tube

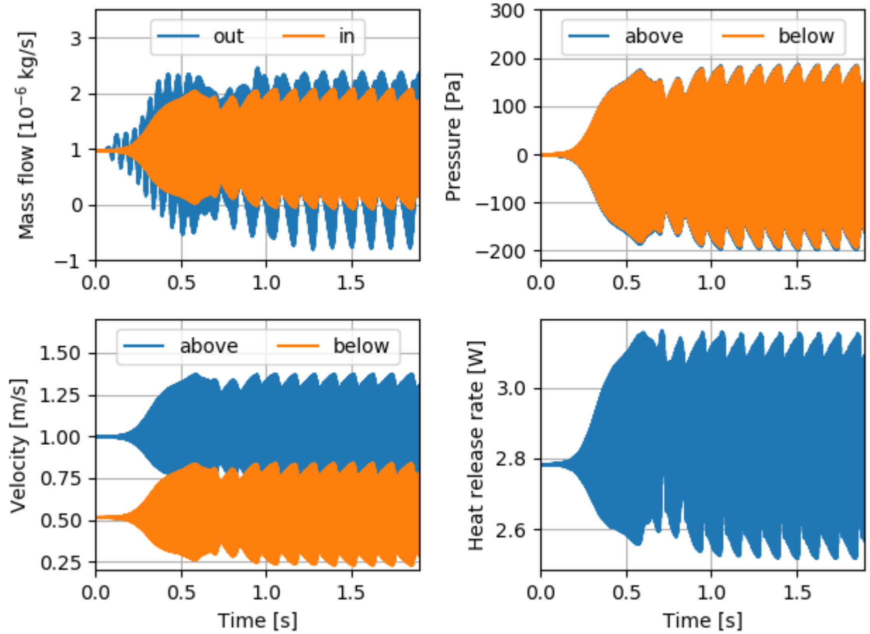
- Also instable at higher temperatures
  - Higher mean flow velocities and heat release rates
  - Higher amplitudes
  - Limit cycle is formed faster
  - Enough energy input to stimulate more frequencies

Temp.	1st harm.
1000 K	482 Hz
1500 K	501 Hz
2300 K	516 Hz

1800 K wire temperature

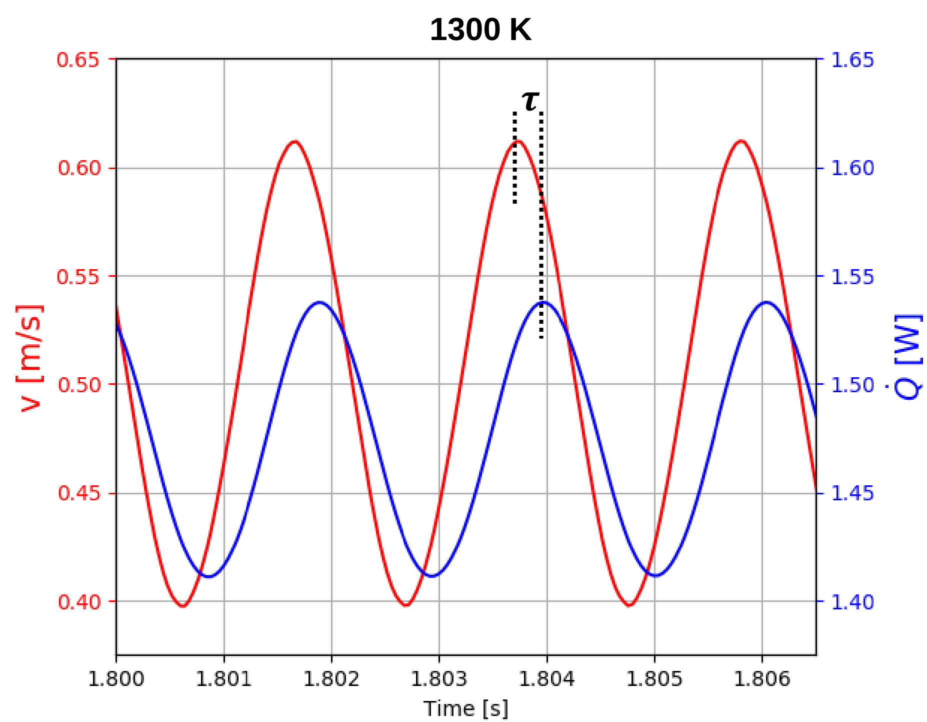


2300 K wire temperature



A Simple Heat Transfer Approximation

- Reduce a complex 3D instability effect to an approximated 1D mathematical context
  - Context: $\dot{Q}(t) \sim v(t)$
    - $\dot{Q}(t)$ is the heat release rate over the wires
    - $v(t)$ is the velocity straight before the wires in pipe-axis direction
  - Decomposition of $\dot{Q}$: $\dot{Q}(t) = \overline{\dot{Q}}(t) + \dot{Q}'(t)$
  - Decomposition of v: $v(t) = \overline{v}(t) + v'(t)$
    - $\overline{\dot{Q}}(t) \sim \overline{v}(t)$
    - $\dot{Q}'(t) \sim v'(t-\tau)$
    - τ – time delay between $\dot{Q}'$ and v'
  - Model: $\dot{Q}(t) = F1 \cdot \overline{v}(t) + F2 \cdot v'(t-\tau)$
    - $F1 = \frac{\overline{\dot{Q}}}{\overline{v}}$
    - $F2 = \frac{\dot{Q}'}{v'}$



F1, F2 and τ Determination

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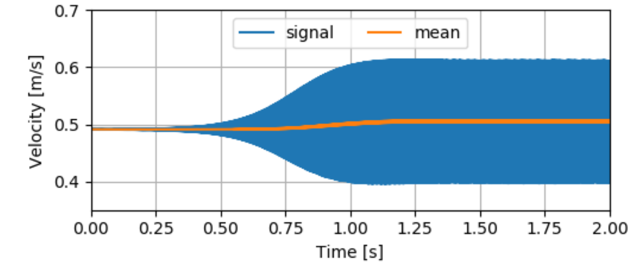
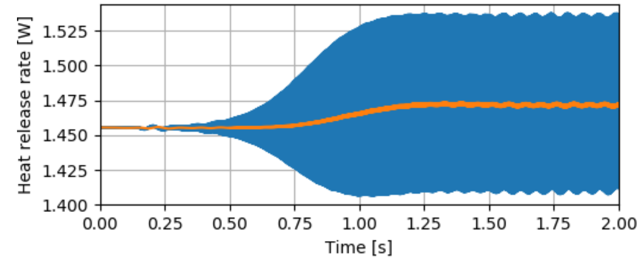
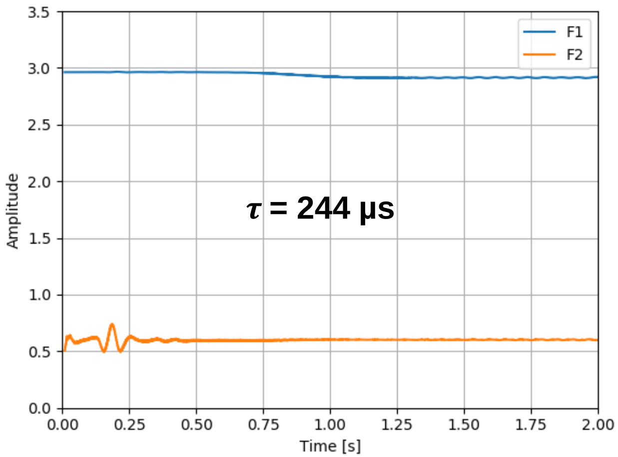
- Time development was observed
  - Stable mean at the beginning (low amplitudes of v' and $\dot{Q}'$)
  - Small change when fluctuation amplitudes grow bigger, due to a mean shift of $\dot{Q}$
  - Stable mean at the limit cycle
  - F1 and F2 are calculated by taking the mean at limit cycle part
  - $\boldsymbol{\tau}$ is calculated via phase angle between v' and $\dot{Q}'$

1300 K

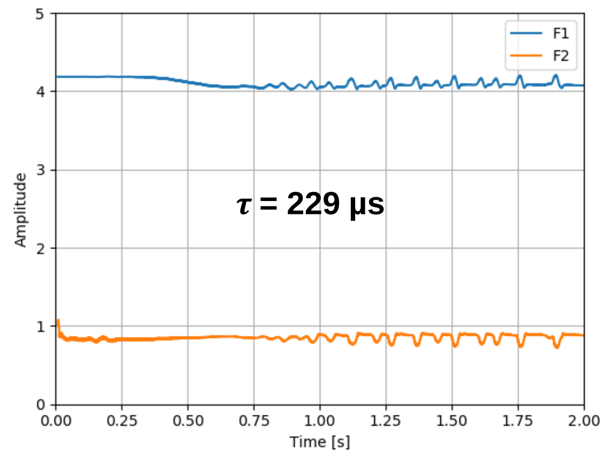




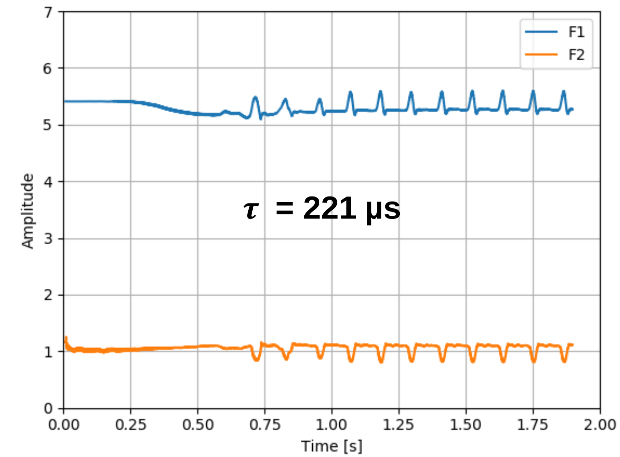
1300 K



1800 K

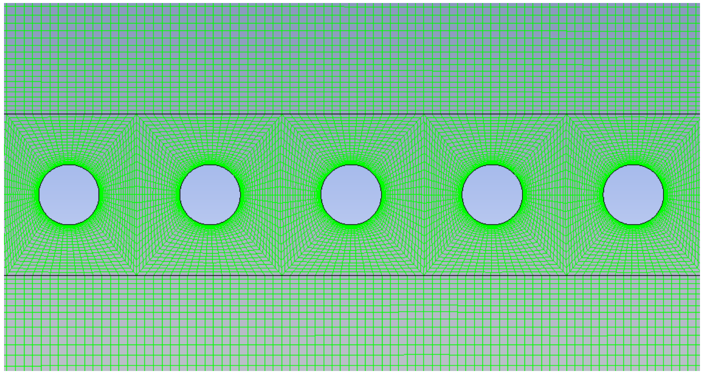


2300 K



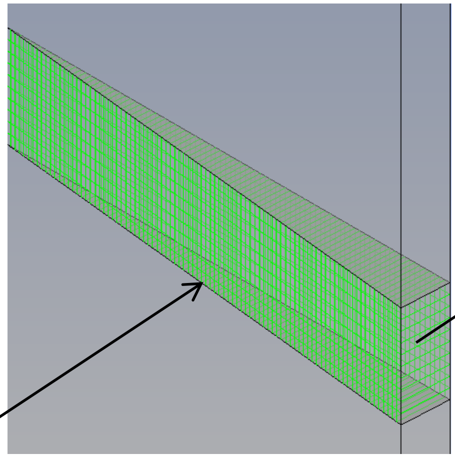
ANSYS CFX Model without Wires

- Wires are replaced by Subdomain
  - Isotropic Loss Model with K_{Loss} set (flow resistance through wires)
  - Total heat source term activated
  - Source: $\dot{Q}(t) = F1 \cdot \overline{v}_{blw}(t) + F2 \cdot v'_{blw}(t - \tau)$
  - v_{blw} - area average at plane below the wires
  - Source defined via User Fortran routine



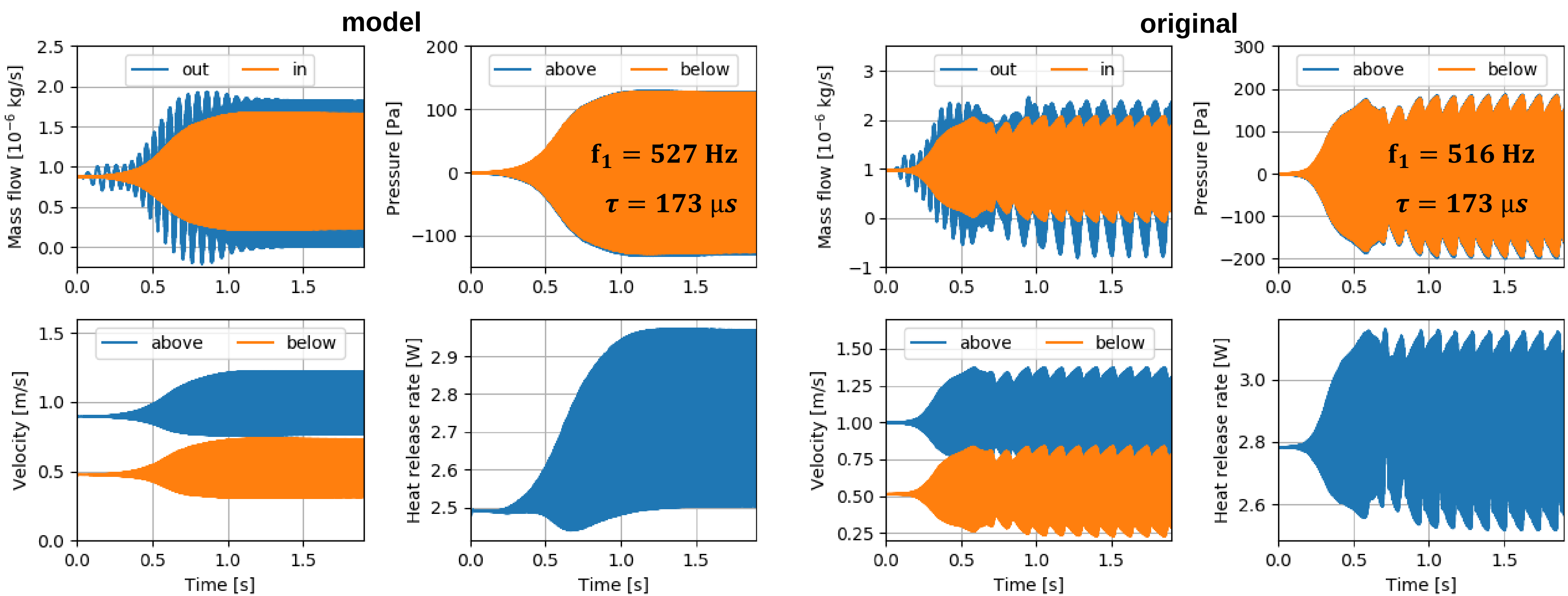
- User Fortran Routine
  - Routine stores v_{blw} for 20 periods
  - Calculates $\overline{v}_{blw}$, v'_{blw}, $\overline{\dot{Q}}$ and $\dot{Q}'$
  - Returns: $\dot{Q}(t) = \overline{\dot{Q}} + \dot{Q}'$

$$\Rightarrow \dot{Q}(t) = F1 \cdot \overline{v}_{blw}(t) + F2 \cdot v'_{blw}(t - \tau)$$



Simulation with Approximated $\dot{Q}$

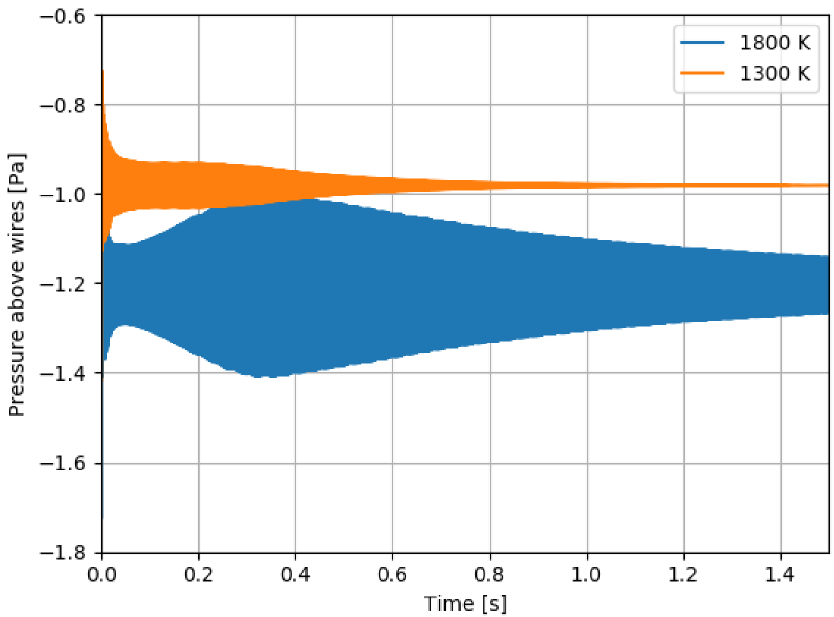
- Model for 2300 K wire temperature works in the expected range
  - Oscillation amplitudes are slightly weaker (1. mode)
  - Resonance frequency is slightly higher
  - Time until limit cycle has formed is longer (different start solutions)
  - Phase delay slightly lower (no distance between v_{blw} and $\dot{Q}$ in Subdomain)



Simulation with Approximated $\dot{Q}$

- Models for 1300 K and 1800 K don't work properly
  - Don‘t self excite ⇒ No limit cycle
  - Variation of different flow/script parameters doesn‘t change the problem
  - Model only works for one specific setup
  - The simple model is too simple

  ⇒ A change in the mathematical model is needed

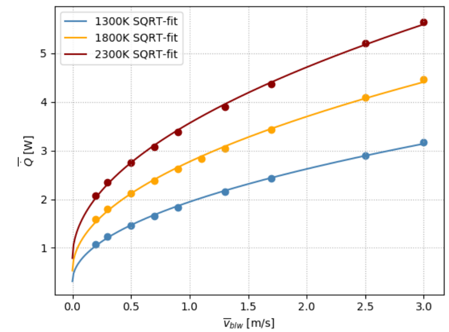


Change in the Mathematical Model

CFX BERLIN

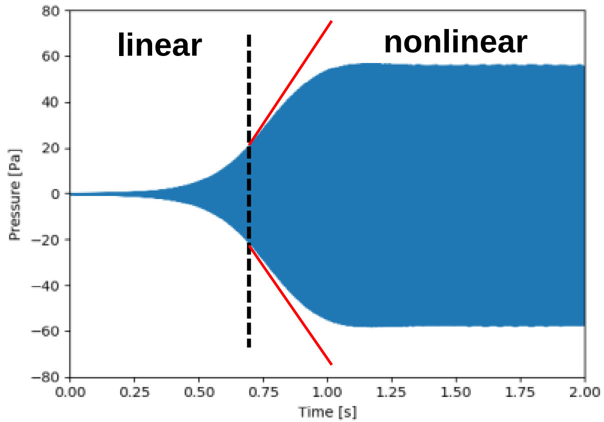
- Unregarded properties of thermoacoustic systems
  - Delay between $\dot{Q}'$ and v' is frequency dependent: $\tau(\omega)$
  - Flames or heated wires have a low pass behavior
  - $\Rightarrow$ Can be represented by a Transfer Function

$f_0(\overline{v}(t))$



- New model
  - New function $f_0(\overline{v}(t)) = \alpha + \beta\sqrt{\overline{v}(t)}$ for $\overline{\dot{Q}}$
  - Model for a Heat Transfer Function $HTF(v'(\omega))$ for $\dot{Q}'$
  - $\Rightarrow \quad \dot{Q}(t) = f_0(\overline{v}(t)) + HTF(v'(\omega))$

- Restriction of Transfer Functions
  - Valid for linear time-invariant systems
  - Only for low amplitudes
  - Transfer functions have no saturation mechanism



HTF Model

CFX BERLIN

- Simulating a Transfer Function for the wires
  - Frequency dependent simulations
  - Called Heat Transfer Function (HTF)

$$HTF(i\omega) = \frac{\dot{Q}'(\omega)}{u'(\omega)} e^{-i\varphi} \cdot \frac{\bar{u}}{\bar{\dot{Q}}}$$

- Calculation of a State Space model (SSM)
  - Representation of the HTF in time domain
  - 6 dimensional state vectors

$$\begin{cases} \dot{\underline{x}}(t) = A\underline{x}(t) + B\dfrac{u'(t)}{\bar{u}} \\ \dfrac{q'(t)}{\bar{q}} = C\underline{x}(t) \end{cases}$$

- Implicit model via User Fortran routine

$$\begin{cases} x^{n+1} = x^n + \Delta t \left[A \cdot x^{n+1} + B \cdot \dfrac{u'^{n+1}}{\bar{u}} \right] \\ \dfrac{\dot{Q}'^{n+1}}{\bar{\dot{Q}}} = C \cdot \underline{x}^{n+1} \end{cases}$$

- Expansion to Advanced SSM (ASSM)
  - Amplitude dependent simulations
  - Leads to an additional nonlinear function
  - $\mathcal{N}\left(\frac{u'(t)}{\bar{u}}\right) = 1 + \gamma_1 \left(\frac{u'(t)}{\bar{u}}\right) + \gamma_2 \left(\frac{u'(t)}{\bar{u}}\right)^2$

$$\begin{cases} \dot{\underline{x}}(t) = A\underline{x}(t) + B\dfrac{u'(t)}{\bar{u}} \\ \dfrac{\dot{Q}'(t)}{\bar{\dot{Q}}(t)} = C\underline{x}(t) \cdot \left(1 + \gamma_1 \left(\dfrac{u'(t)}{\bar{u}}\right) + \gamma_2 \left(\dfrac{u'(t)}{\bar{u}}\right)^2\right) \end{cases}$$

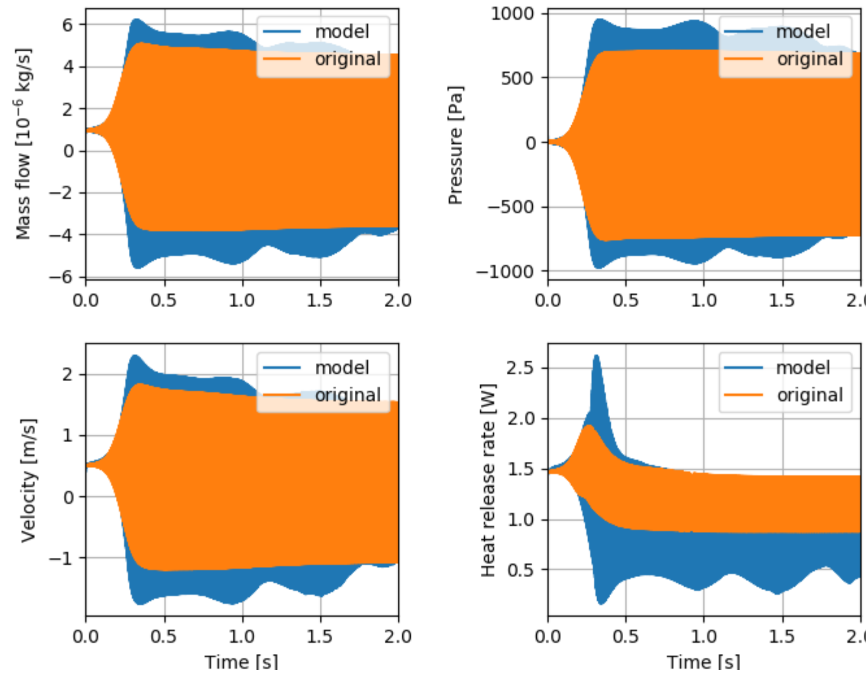
Simulating the Self Exciting case with ASSM

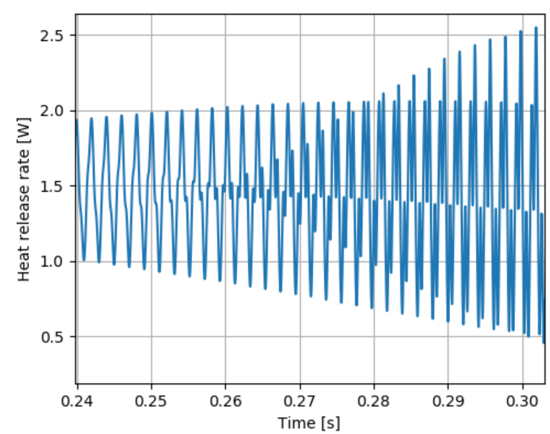
- System self excites as expected
  - Amplitudes are slightly higher
  - Limit cycle not completely stable
  - Resonance Frequency (1. mode):
    - Model 527 Hz (up to 550 Hz)
    - Original 527 Hz
    - FFT after 1.52 s
- 3rd mode gets amplified strongly
  - after 0.26 s
  - Could be a model problem of the ASSM:

$$\dot{\underline{x}} = A\underline{x} + B \cdot u$$
$$y = C\underline{x} + \gamma_1 C\underline{x} \cdot u + \underline{\gamma_2 C\underline{x} \cdot u^2}$$

$u \sim \sin(\omega) \Rightarrow$ Integrating $sin^3(\omega)$ leaves a $\cos(3\omega)$ term $\Rightarrow$ 3rd harmonic

1300 K





Summary and Outlook

- Simple model only works for specific flow parameters
  - Easy to determine
  - Doesn’t cover frequency dependencies of thermoacoustic effects

- ASSM model covers more thermoacoustic characteristics
  - More complex to determine
  - 30 simulations for the final HTF and non linear function each
  - New problems appear (3rd harmonic)

- Saving computational power leads to lower simulation times
  - Mesh with wires ≈34000 elements ⇒ 3 days on 4 CPU Cores
  - Mesh without wires ≈13000 elements ⇒ 2 days on 2 CPU Cores

- Outlook
  - Implementation into 1D CFD would reduce simulation time even more
  - Usage of simplified models to replace experiments ⇒ less time and cost

Questions?

Thank you for your attention

For any further questions feel free to contact me:
paschke.dennis@gmx.de

HTF Model Determination

CFX BERLIN

- Simulating a Transfer Function for the wires
  - Frequency dependent simulations
  - Different wire temperatures investigated
  - Called Heat Transfer Function (HTF)

$$HTF(i\omega) = \frac{\dot{Q}'(\omega)}{u'(\omega)} e^{-i\varphi} \cdot \frac{\bar{u}}{\bar{\dot{Q}}}$$

- Calculation of a State Space model (SSM) for the Transfer Function
  - Model transformation into time domain

State Space Model
$$\begin{cases} \dot{\underline{x}}(t) = A\underline{x}(t) + B\dfrac{u\prime(t)}{\bar{u}} \\ \dfrac{q\prime(t)}{\bar{q}} = C\underline{x}(t) \end{cases}$$

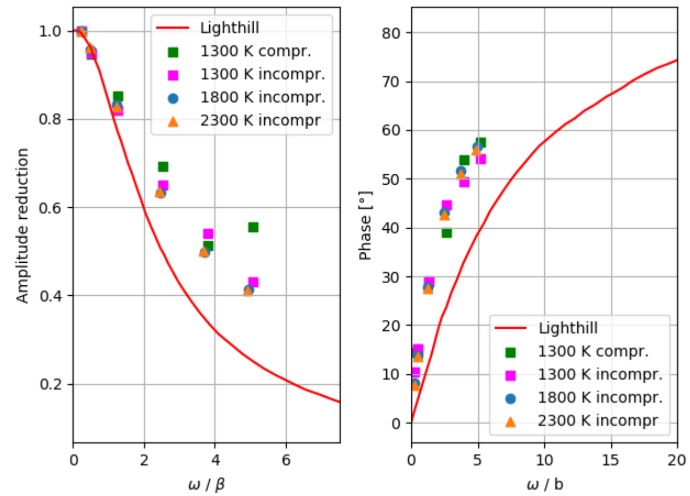
  - Feeding “Vector Fit” with discrete frequency dependent data leads to 6 dimensional state vectors (https://www.sintef.no/projectweb/vectfit/)

- Implicit model via User Fortran routine

$$\begin{cases} x^{n+1} = x^n + \Delta t \left[A \cdot x^{n+1} + B \cdot \dfrac{u'^{n+1}}{\bar{u}} \right] \\ \dfrac{\dot{Q}'^{n+1}}{\bar{\dot{Q}}} = C \cdot \underline{x}^{n+1} \end{cases}$$

Determination of and Simulation with the HTF

- Determined HTF compared to Lighthills theory
  - Amplitude reduction $\Rightarrow \dfrac{\dot{Q}'(\omega)/v'_{blw}(\omega)}{\dot{Q}'(\omega_0)/v'_{blw}(\omega_0)}$
  - Results show same trend
  - 24 simulations needed for the presented data **only**

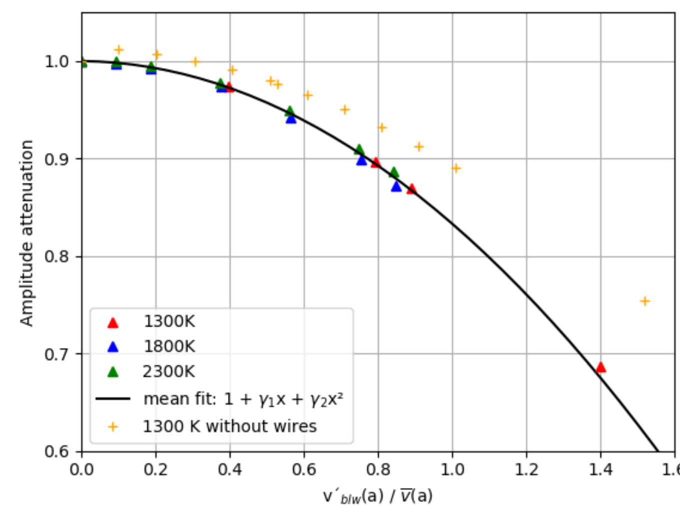


- The Nonlinear Function (saturation)
  - Amplitude dependent simulations $\Rightarrow \dfrac{\dot{Q}'(a)/v'_{blw}(a)}{\dot{Q}'(a_0)/v'_{blw}(a_0)}$
  - One function for all temperatures:

$$\Rightarrow \mathcal{N}\left(\frac{u'(t)}{\bar{u}}\right) = 1 + \gamma_1\left(\frac{u'(t)}{\bar{u}}\right) + \gamma_2\left(\frac{u'(t)}{\bar{u}}\right)^2$$

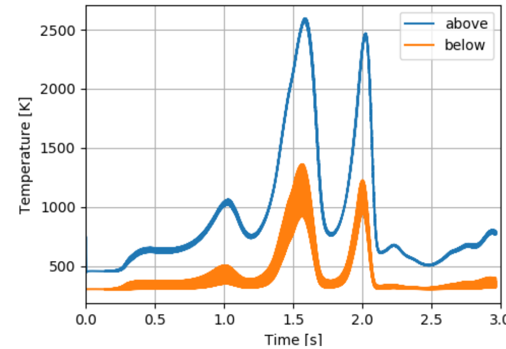
  - Implemented via User Fortran routine

$$ASSM(t,a)\begin{cases} \dot{\underline{x}}(t) = A\underline{x}(t) + B\dfrac{u'(t)}{\bar{u}} \\ \dfrac{\dot{Q}'(t)}{\bar{\dot{Q}}(t)} = C\underline{x}(t)\cdot\left(1+\gamma_1\left(\dfrac{u'(t)}{\bar{u}}\right)+\gamma_2\left(\dfrac{u'(t)}{\bar{u}}\right)^2\right) \end{cases}$$

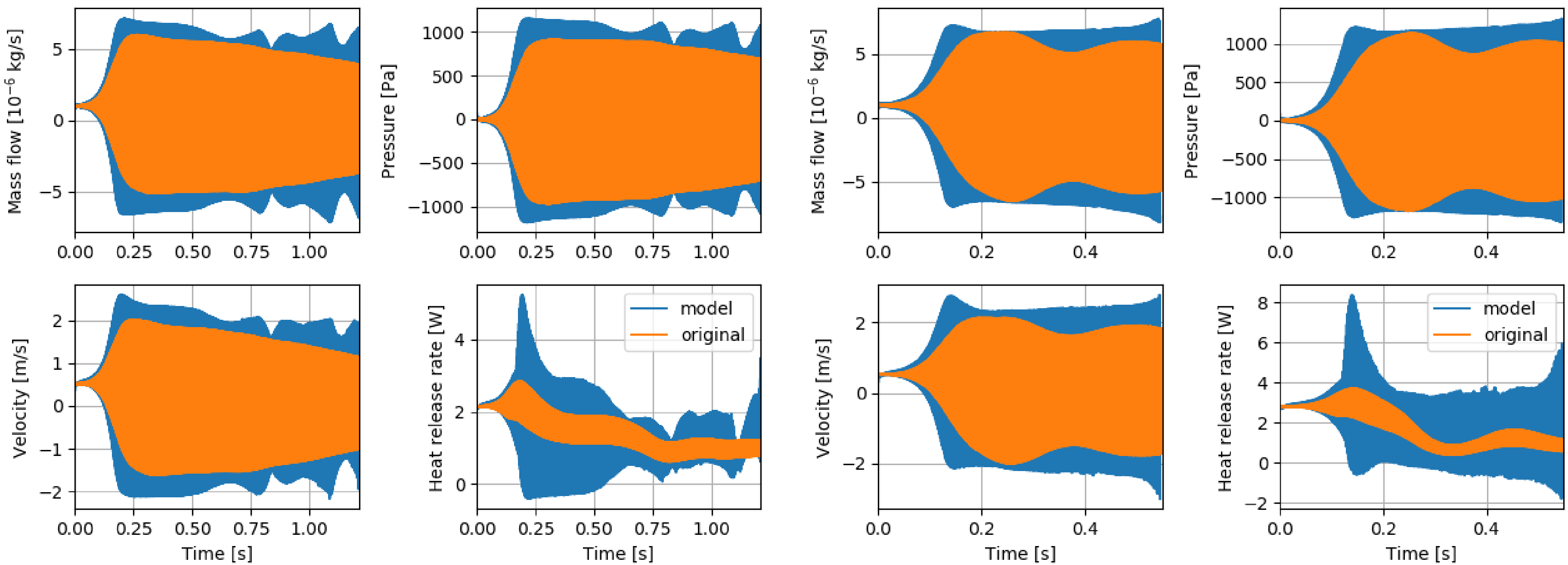


ASSM at 1800 K and 2300 K

- System self excites as expected
  - Amplitudes are slightly higher
  - Temperature increase too strong
  - At 1300K it’s still high but doesn’t explode
  - Temperature explodes at higher wire temperatures



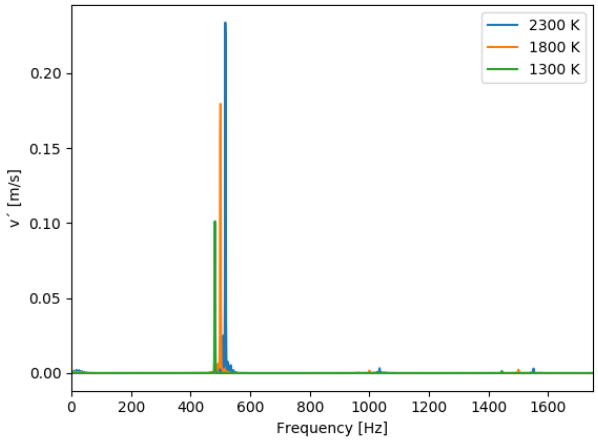
1800 K **2300 K**

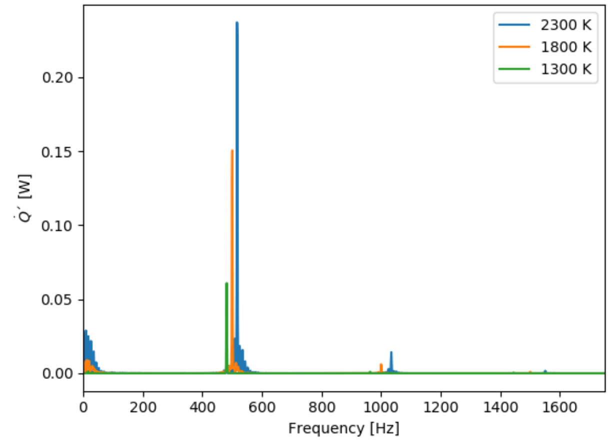


Fourier Analysis of the Rijke Tube

CFX BERLIN

- Resonance frequencies are excited
  - Clean spectrum of the velocity fluctuation straight below the wires
  - $\dot{Q}'$ also oscillates at the 2nd harmonic (and at very low frequencies)
  - Calculated frequencies are slightly higher
  - End correction was neglected in the calculation

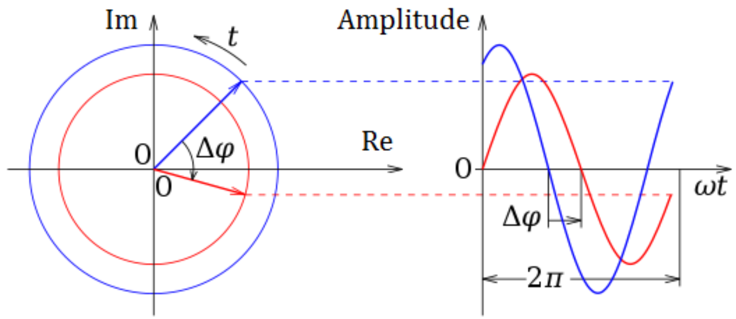




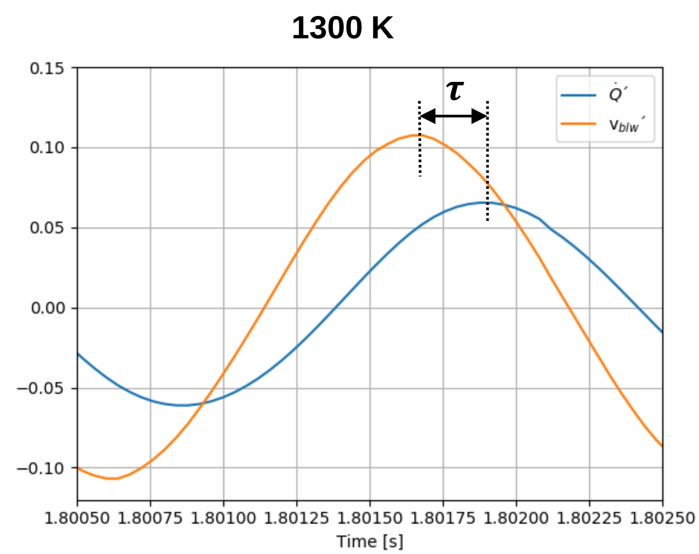
Parameter	Value				Unit
ΔT	0	1000	1500	2000	K
$f_{1,calc}$	434	489	511	529	Hz
$f_{1,sim}$	-	482	501	516	Hz

Determination of τ

- Calculating phase angle difference
  - Using FFT data of v' and $\dot{Q}'$
  - Calculate phase delay φ
  - $\boldsymbol{\tau} = \frac{\Delta\varphi}{\omega} = \frac{\Delta\varphi}{2\pi \cdot f_1}$



Parameter	Value			Unit
$\Delta \boldsymbol{T}$	1000	1500	2000	K
$f_{1,sim}$	482	501	516	Hz
$\Delta\varphi$	42.32	41.39	41.02	°
$\boldsymbol{\tau}$	244	229	221	µs

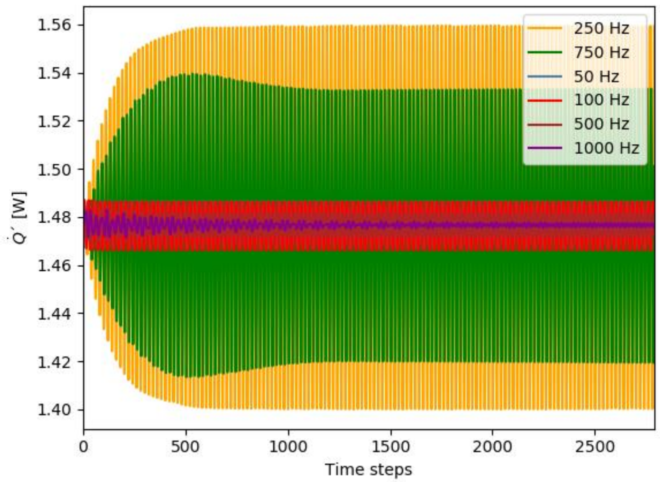


Determination of the HTF

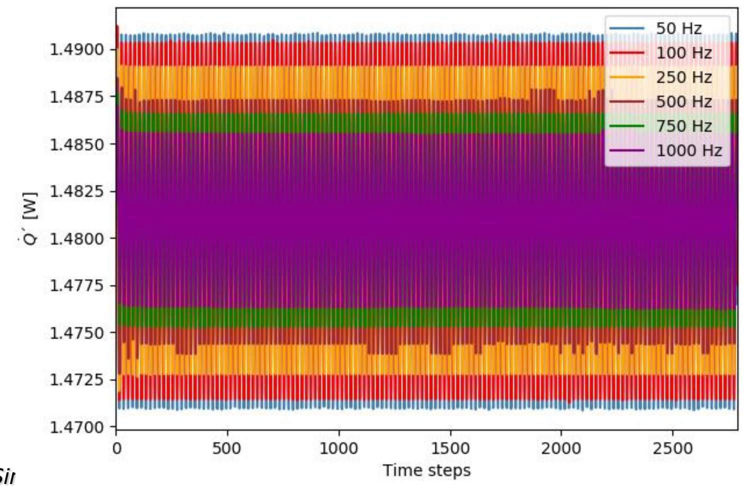
- Problem with compressible air model
  - At 250 and 750 Hz the oscillation amplitude gets amplified
  - Maybe a problem with the systems acoustics
  - Bypassing by using incompressible air

- Simulations with incompressible air show expected behavior
  - Density can still change when the temperature changes (Table Generation)
  - Heat Transfer Model: Thermal Energy

Compressible (1300 K)



Incompressible (1300K)



Final Heat Transfer Model

- Derivation

$$\dot{Q}(t) = \overline{\dot{Q}}(t) + \dot{Q}'(t)$$

$$\dot{Q}(t) = f_0\big(\overline{u}(t)\big) + AHTF(\omega, a) \qquad \text{with} \quad f_0\big(\overline{u}(t)\big) = \alpha + \beta\sqrt{\overline{u}(t)}$$

$$\dot{Q}(t) = f_0\big(\overline{u}(t)\big) + HTF(\omega) \cdot \mathcal{N}(a) \qquad \text{with} \quad \mathcal{N}(a) = \mathcal{N}\left(\frac{u'(t)}{\overline{u}}\right) = 1 + \gamma_1\left(\frac{u'(t)}{\overline{u}}\right) + \gamma_2\left(\frac{u'(t)}{\overline{u}}\right)^2$$

$$HTF(\omega) \Rightarrow SSM(t) \begin{cases} \dot{\underline{x}}(t) = A\underline{x}(t) + B\,\dfrac{u'(t)}{\overline{u}} \\ \dfrac{\dot{Q}'(t)}{\overline{\dot{Q}}(t)} = C\underline{x}(t) \end{cases}$$

$$AHTF(\omega, a) \Rightarrow ASSM(t, a) \begin{cases} \dot{\underline{x}}(t) = A\underline{x}(t) + B\,\dfrac{u'(t)}{\overline{u}} \\ \dfrac{\dot{Q}'(t)}{\overline{\dot{Q}}(t)} = C\underline{x}(t) \cdot \left(1 + \gamma_1\left(\dfrac{u'(t)}{\overline{u}}\right) + \gamma_2\left(\dfrac{u'(t)}{\overline{u}}\right)^2\right) \end{cases}$$